Precise determination of the refractive index of air in Fabry-Perot cavity by means of the optical frequency comb

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INTRODUCTION

Optical frequency metrology mostly deals with frequency synthesis through pulsed femtosecond mode-locked lasers in these days [1]. The value of repetition rate of generated pulses determines (in the frequency domain) spacing of discrete coherent spectral components of the whole supercontinuum [2, 3]. The broad set of components in the supercontinuum is referred as an optical frequency comb and the whole pulsed laser system generating the stable comb spectrum can be considered as an optical frequency synthesizer. It is able to transfer relative stability of radio frequency (RF) pulse repetition frequency into optical spectral domain and vice versa. Therefore the synthesizer can be considered an advanced metrology tool bridging a large frequency gap between optical and radio frequency bands. The main application of the synthesizer may be a direct comparison of relative stabilities of ultra-stable RF generators based on microwave atomic clocks (i.e. Rb, Cs) and optical frequency standards such as lasers stabilized through spectroscopy of molecular, atomic, or ions transitions (i.e. I_2, Yb^+ or Hg^+) [4]. The relative stability of optical frequency standards can be measured by comparison with the stabilized femtosecond laser comb [5]. The definition of the one meter is based on the path travelled by light in vacuum. The distance is then derived from stabilized optical frequency of lasers where wavelength is linked to time through the speed of light [6]. Counting of wavelengths and practical displacement measurements are a task for interferometers. Direct conversion of the optical frequency into mechanical length is based on locking of an optical cavity to a laser at resonance frequency [7]. Stabilized mode-locked lasers represent sources of pulses with very stable spacing. In the field of nanometrology the quest for even higher resolution and precision may lead to the subnanometer and picometer range [8, 9]. The Fabry-Perot cavity (FPC) combined with the femtosecond laser synthesizer optical frequency comb referenced to RF standard could lead to precise calibration of astronomical spectrographs [10] or in precision spectroscopy [11]. In this paper we present monitoring of spacing of Fabry-Perot mirrors in ambient air by the use of mode-locked femtosecond laser comb. Changes of optical path distance are monitored through changes in repetition frequency of pulses. With measurement of absolute mirror distance in vacuum chamber, absolute refractive index of air can be defined.

FABRY-PEROT CAVITY AND FREQUENCY COMB

Optical frequency combs are based on laser sources generating a train of the femtosecond pulses characterized by the central wavelength, repetition frequency of the pulses, pulse shape and pulse to pulse phase shift [1] with frequency spectrum:

$$f_i = f_{ceo} + i \cdot f_{rep} \qquad (1)$$

where i is the number of comb lines in the order of 10^6 and f_{rep}, and f_{ceo} are repetition and offset frequency in the RF domain, respectively. Stabilization of optical frequency f_i is based on stabilization of f_{rep}, and f_{ceo} frequencies by the second harmonic generation in nonlinear crystal [12, 13]. On the other hand any instability represented in optical domain is transferred into the instability in f_{rep} and f_{ceo}. Thus offset and repetition frequencies can be separated in a comb with octave spanning and locked to any RF standard (such as atomic clock). RF atomic clocks transfer its stability into optical domain or they could be used as ultra-stable RF reference measuring changes of repetition frequency caused by changes in optical domain.

Fabry-Perot cavity (FPC) consists of two mirrors with spacing L_{cav}. It operates as a highly selective optical filter performing narrow-band transmission spectrum with a repetition frequency related to the optical path distance (OPD). The FPC's free spectral range (FSR):

$$\nu_{opt} = j \cdot \nu = \frac{c}{L_{opd}} \qquad (2)$$

where c: speed of the light, j: integer number, L_{opd}: optical length related to the spacing of the cavity mirrors, where $L_{opt} = 2nL_{cav}$ or $L_{opt} = 4nL_{cav}$ for confocal configuration and n is the refractive index of the environment. Transmitted optical resonance frequencies fulfill following condition (in confocal configuration):

$$f_m = \frac{mc}{4nLcav} \tag{3}$$

For a single spectral component of a comb at coincidence with resonance frequency of the FPC (f_{m}), optical frequency of the laser comb follows this condition:

$$f_i = f_m = f_{ceo} + i \cdot f_{rep} = \frac{mc}{4nL_{cav}} \tag{4}$$

and defines OPD:

$$L_{opt} = 4nL_{cav} = \frac{mc}{(f_{ceo} + i \cdot f_{rep})} \tag{5}$$

OPD (5) covers information of :
1. refractive index of air:

$$n = \frac{mc}{4L_{cav}(f_{ceo} + i \cdot f_{rep})} \tag{6}$$

So precise monitoring of refractive index of ambient air need use of FPC with ultra-stable mirror spacer based on ultra low expansion materials with good temperature stabilization and other for example well chosen mechanical stability of FPC.
2. exact cavity length in vacuum:

$$L_{cav} = \frac{mc}{4(f_{ceo} + i \cdot f_{rep})} \tag{7}$$

Precise monitoring of the mirror distance of the FPC and precise monitoring of expansion ultra-stable mirror spacer requires special conditions- such as well mechanically stabilized and evacuated chamber.

The mirror distance L_{cav} depends according to (7) on f_{ceo} and on f_{rep} where m and i are integers. The L_{cav} (or nL_{cav} in ambient air) stability is linked to the optical frequency comb component. The optical frequency comb component stability is referenced multiplicatively to the repetition frequency (f_{rep}) and additively to offset frequency f_{ceo}. The distance between two mirrors of FPC is indirectly proportional to optical frequency transmitted into the FPC. Locking the optical frequency to the FPC leads to direct transfer of OPD changes into the optical frequency domain. Optical frequency of separated optical frequency comb component (4) then transfers to the changes of the repetition frequency while the offset frequency is stabilized to RF referenced clock.

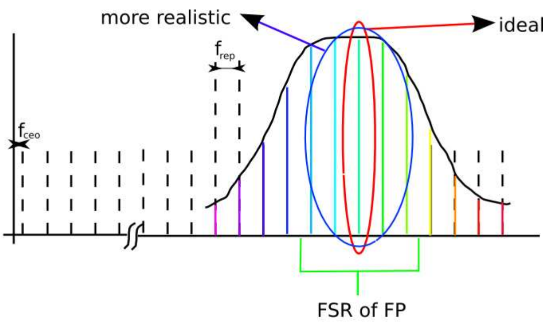

Fig. 1. Schematics of narrow-band optical grating selectivity of optical frequency comb. FSR: free spectral range, FP- Fabry-Perot cavity, f_{rep} and f_{ceo}: repetition and offset frequency, respectively.

MONITOR OF FABRY-PEROT OPTICAL PATH DISTANCE

Using the full frequency comb spectra leads to interferogram [14] which centroid consists information about the length of FPC, beam shape and dispersion in optical path distance. The scheme of the comb component separation is in Fig. 1 [15]. In the ideal situation only one optical frequency is transmitted. While one component of the femtosecond mode-locked laser is locked to the FPC length then according to (5) in air or (7) in vacuum the repetition and offset frequencies represent mirror distance changes. Locking the repetition frequency f_{rep} and offset frequency f_{ceo} to RF standard and filtering in optical domain one component of femtosecond laser bomb acts as any ultra-stable single-frequency laser standard which might be used in measuring of lengths [6].
The narrow band filtering is here the main task. For a FSR of approx. 2 GHz which corresponds to the FPC with 37.5 mm length it is necessary to get a filter with the bandwidth better than a several picometers in optical wavelength domain. Free space ruled blazed diffraction grating in monochromator configuration lead only about 0.1 nm for 1 m long monochromator. For that reason we chose to seperate the wavelength by fiber Bragg grating (FBG) with FWHM of less than 0.1 nm of reflected light.

In our pilot experiment [16] we used the stabilized femtosecond mode-locked laser with 100 MHz repetition frequency working at 1550 nm central wavelength (MenloSystems, GmbH). It is a fiber based mode-locked laser. Using of fiber optical parts is advantageous as well as using of FBG. The FBG operate in a reflective regime. In Fig. 2 is the output of the femtosecond laser connected to a fiber circulator (port no. 1). The circulator prevents reflection of the laser light from other components connected to ports no. 2 and no. 3. Light beam passes through the circulator directly to the port no. 2 and the light from the narrow-band FBG is reflected to the port no. 3. Optical output on this port consists of several spectral components of the source femtosecond laser. They are delivered by fiber and collimator into the FPC. We have designed a confocal cavity with FSR 2 GHz and finese 339 (quality factor of $3.3 \cdot 10^7$ at 1547 nm).

In our pilot experimental setup we used ultra-narrow and very high reflective FBG from O/Eland, Inc. It has the central wavelength 1547.198 nm and FWHM of 0.017 nm and reflectivity of 95.019%. Such a grating reflects a spectral range 2.1 GHz from the broad comb spectrum. It contains approx. 21 frequency components transmitted through the FPC by a collimator. It means that the laser components at the edge of the relected spectrum still overlap with another edge components in FPC output spectrum. It can be expected that in the center of each group of output spectrum components there will be some lines with a little overlapping from the fringes of the neighboring group of components. The mirror spacer of the FPC was made from an ultra low expansion ceramics (Zerodur). The light filtered by the circulator and fiber grating (output port no. 3) is also partially directed by a fiber coupler 50/50 into wavelength-meter based on Fizeau interferometers. It has 30 MHz resolution and it is used for monitoring of the central wavelength of separated components by the FBG. Femtosecond laser repetition frequency f_{rep} and offset frequency f_{ceo} are monitored by counters stabilized to GPS reference. Stabilization of f_{ceo} is done by the self-referencing method with using a non-linear interferometer and SHG crystal (technique of the frequency comb) [1].
The infra-red optical signal is detected by photodiode detector (PDI) and fed into a servo loop used to lock the selected longitudinal mode of the cavity to one chosen frequency comb component. For monitoring of refractive index of air we monitor changes in refractive index as a change of repetition rate frequency and offset frequency. Thus the servo-loop controls the repetition frequency of the femtosecond laser and is linked to the cavity length and femtosecond mode-locked laser tracks the FPC optical path distance changes. Signals of spectrum analyzer and by counter Agilent 53123A connected through GPIB bus to the computer. The computer controls whole pilot experimental setup through program running under Labview environment.

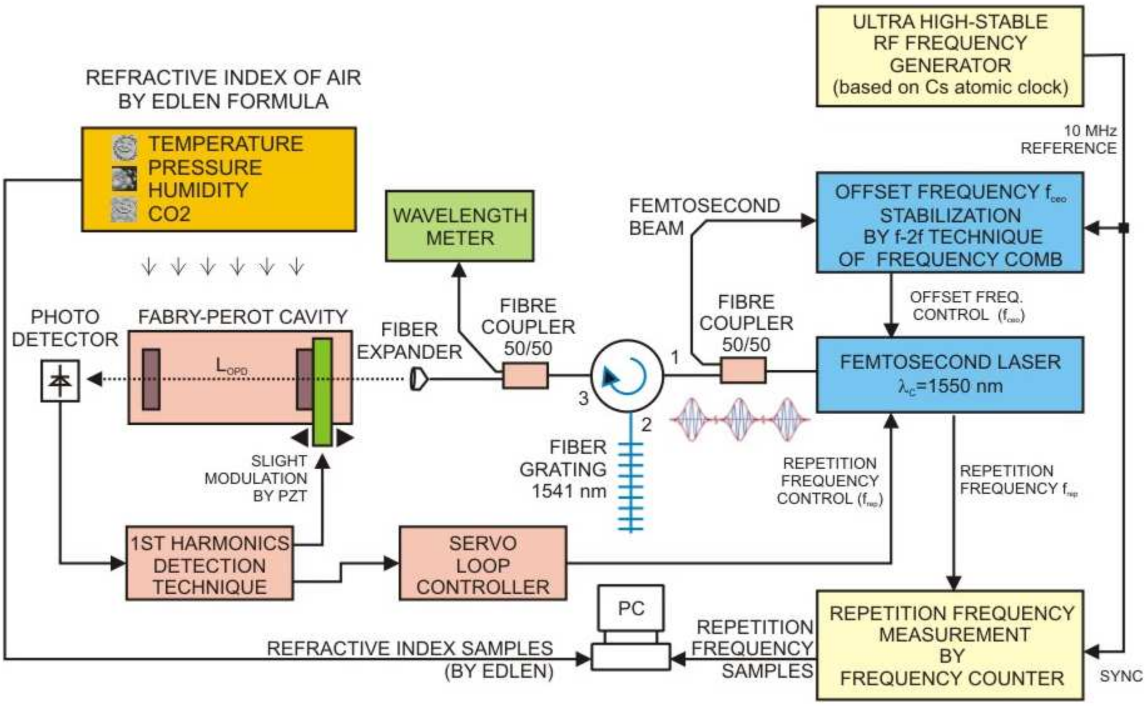

Fig. 2. Scheme of setup for monitor of Fabry-Perot cavity optical path distance by femtosecond laser comb by the use of ultra-narrow optical band fiber Bragg grating and comparison with changes of refractive index of air from Edlen formula. Modulation of Fabry-Perot cavity was not used in the case of 400 MHz Zerodur mirror spacer cavity.

Changes of refractive index of air were measured by the use of Zerodur mirror spacer FPC (Fig. 3) with 400 MHz FSR. Loop based on signal from photodetector controls the repetition frequency of femtosecond comb by first harmonics detection technique for chosen optically separated and throught FPC transmitted femtosecond comb component.

RESULTS

Typical recordings of the transmission spectrum of the FPC is in Fig. 4 for the wavelength region 1547 nm. The spectrum at 1547 nm was filter by narrow-band fiber optical grating from the femtosecond mode-locked laser.

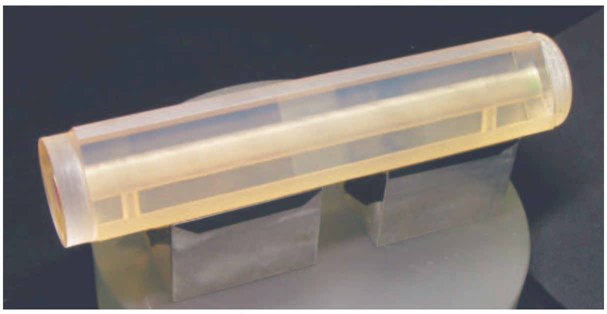

Fig. 3: Photo of the Fabry-Perot cavity with 400 MHz free-spectral range based on Zerodur spacer and fused silica mirrors.

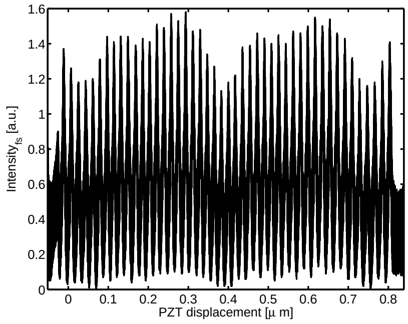

Fig. 4. Recordings of the intensity of fs laser signal in full FSR of Fabry-Perot cavity for full PZT range.

The spectrum represents more than 2 FSR of 2 GHz (=4 GHz) so 40 frequency comb lines at 1547 nm are presented. We observe 20 frequency comb lines the spectral pattern for one FSR of FPC at 1547 nm (Fig. 4) and this pattern repeats twice within the tuning range of the PZT. According to early assumption it is clear that there is a spectral overlap at the fringes of the group of spectral components but due to the relatively high FSR of the FPC there are components at the center of each FSR quite free from aliasing. One of the central components of FSR of frequency comb was chosen for measuring of relative changes of refractive index of air. Fabry-Perot cavity was kept in ambient air and closed in plexiglas box closing and avoiding fast changes from air-conditioning system but perforated for information about current refractive index of air in the laboratory.

Measurement of refractive index of air is presented in Fig. 6. Tracking the optical path distance of FPC to optical component of frequency comb was measured by changes in repetition frequency by means of counter referenced to atomic clock reference. Top part of Fig. 6 represents measurement by FPC and femtosecond laser comb. Absolute value of $n_{FPC} - 1$ was calculated from Edlen equation using few samples in the beging of the measurement. Lower part represents the results obtained by undirect Edlen method. The absolute diference between these two measurements is $3.0 \cdot 10^{-7}$ and the standard deviation between these two measurement is $2.0 \cdot 10^{-7}$. These results of pilot experiment are very promising for further research. Comparison among the measurements by previously presented method with FPC and He-Ne lasers [17] recent method and method based on classical Edlen's method calculated from known temperature, humidity, pressure and CO_2 concetration of ambient air is shown in Fig. 7. Recent method manifest Berger sensitivity to slow changes of refractive index of air such as pressure, humidity and concentration of CO_2.

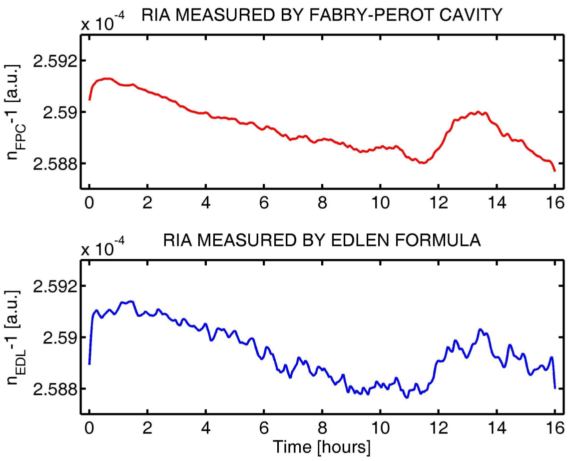

Fig. 6. Results from measurement of changes in refractive index of air by Edlen method and by separated component of femtosecond laser comb.

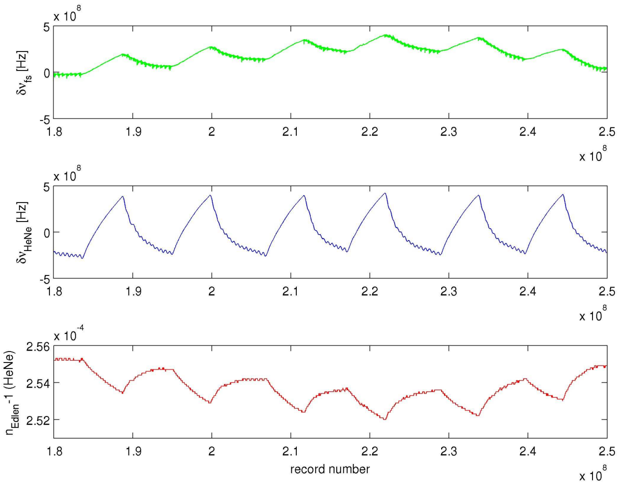

Fig. 7. Comparison among the measurement based on He-Ne lasers [17], frequency comb and classical Edlen's method calculated from known temperature, humidity, pressure and CO_2 concetration of ambient air.

CONCLUSION

We experimentally demonstrated a method for monitoring of cavity length changes based on femtosecond laser comb. Our pilot experiment based on separation of frequency comb lines by fiber Bragg grating produces an unlimited variety of Fabry-Perot cavity lengths to be locked on. We experimentally demonstrated that the Fabry-Perot cavity length could be locked to one chosen femtosecond frequency comb line. Tracking the optical path distance of FPC to optical component of frequency comb can be directly measured by means of counter of repetition frequency.

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